Discovering Rotor Noise

The rotor is turning at an angular velocity Ω_0 = 6000RPM. For a 1-blade rotor, the time needed to do a complete turn of 2π radians is given by $T_0 = 2\pi/\Omega_0$ which is 0.01s and the frequency (f_0) associated with this period is 100 Hz. Similarly, for a rotor with N blades equally distributed in the azimuthal direction every $\Delta\theta = 2\pi/N$, the time needed to turn the system by $2\pi/N$ radians is the fundamental period T_0 of the system. The frequency associated with this period is called the Blade Passing Frequency (BPF).

Blade Passing Frequency (BPF) $= 1/T_0$; where $T_0 = (2\pi/N)/\Omega_0$ .... (1)

Therefore, from the eqn.(1), for a 2-blade rotor turning at 6000 RPM, the BPF is 200 Hz.

1. Computing the FFT:

The FFT of the pressure amplitude is done using a *fftpack* in python. Performing the FFT on a large dataset requires a lot of memory. Dividing the data into smaller bloks allows to perform FFT on each blok separately, which can be less memory-intensive. Normalizing the FFT is important for making the amplitude values of FFT more meaningful and for comparing FFTs.

As suggested during the course, to display the Fourier spectrum in the frequency domain, the associated frequency f_m of the indices *m* needs to be computed and $f_m = 0$ to $f_s/2$ are useful, where f_s is the sampling frequency. Hence, the maximum frequency that can be obtained is dependent on the sampling time T_s at which samples are obtained. The useful samples $N = T_{end}/2T_s = N_s/2$ should be normalized.

One common way is to divide the amplitude values by the number of samples. The normalization factor is 1/N. In our case, it becomes $2/N_s$. This means that the amplitude values of the FFT would be divided by $N_s/2$ to get the normalized FFT that would have amplitude values that are proportional to the amplitude values of the original signal.

Another way is using RMS (Root Mean Square) which involves dividing the amplitude values of the FFT by the RMS value of the original time-domain signal. For example, if we have a time-domain signal x(n) with N samples ($N_s/2$ in our case), the RMS values if the signal would be calculated as:

$$RMS = sqrt(sum(x(n)^2)/N \qquad .... (2)$$

This RMS method has been used in the python code as normalization for length and P_{rms}.

For real-valued signals (such as acoustic pressure in our case), the negative frequencies are just complex conjugates of the positive frequencies and carry redundant information. Therefore, only positive frequencies can be used to compute the amplitude spectrum. When computing such one-sided spectrum, the resulting values are only half of the true amplitudes and to recover the true amplitude values, we need to multiply by a normalization factor of 2. So, using a coefficient of 2 instead of A/2 is necessary to recover the correct amplitude values and this normalization of the one-sided spectrum has been implemented in the python code.

SIEMENS is using a "Hanning window" when computing the SPL. Same windowing method has been implemented in the code. When applying a windowing process, the resulting spectrum will be affected by a loss of amplitude. Hanning window tapers the signal at the edges (see Fig.2), reducing the contribution of the signal values near the beginning and end of the time window.

The Boxcar window: $\omega(t) = 1$ for $0 < t < T$ ...... (3)

The Hanning window: $\omega(t) = \frac{1}{2} - \frac{1}{2}\cos(2\pi/T)$ for $0 < t < T$ ...... (4)

A correction factor of 2.0 to the amplitude values in the resulting spectrum must be applied. This is from the fact that the Hanning window has an energy reduction of half compared to rectangular window of same length.

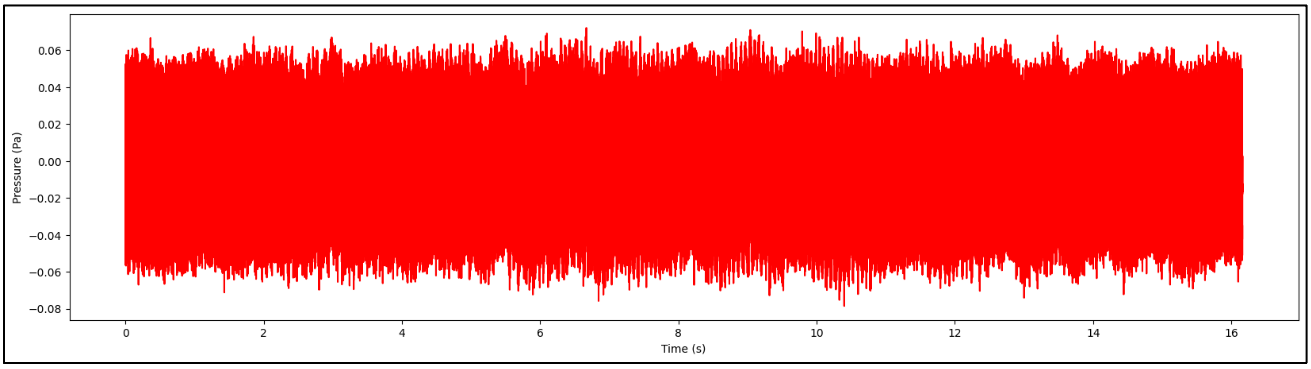

Figure.1 Pressure amplitude in time.

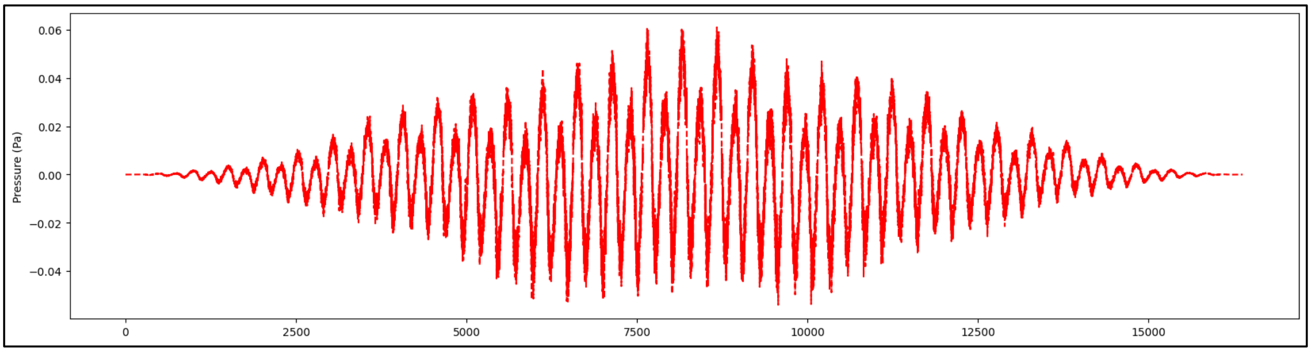

Figure.2 Pressure amplitude in frequency after applying Hanning window.

2. Trade-off for choosing no: of bloks:

As aforementioned, by dividing the data into bloks makes the FFT algorithm more efficient. However, using multiple blocks for FFT can also have some trade-offs. The FFT assumes that the data is periodic, and if the bloks are not aligned with the periodicity of data, there can be overlap between bloks and this can introduce spectral leakage and other artifacts.

Another trade-off to consider is that smaller bloks can reduce the frequency resolution of the FFT output. This is because the frequency resolution (Δf) of FFT is inversely proportional to the length of the dataset.

$$\Delta f = f_s/N \quad \text{...... (5)}$$

Generally, the number of bloks are chosen so that the bloks are long enough to preserve the frequency content of the data but short enough to be computationally efficient. SIEMENS has cut the initial signal into $N_{blok} = 49$ parts. The FFT is computed for each blok, and final FFT modulus value is obtained as the mean over these bloks.

$$FFT = \frac{1}{N_{blok}} \sum_{k=1}^{N_{blok}} |FFT_k| \quad \text{...... (6)}$$

Same number of bloks are used in computation as the final output is compared with the siemens spectrum.

3. Computing and analysing the SPL spectrum:

The output of FFT is converted into SPL spectrum by multiplying the output with normalization factors that depends on the sampling frequency and the length of the signal as mentioned in the above section.

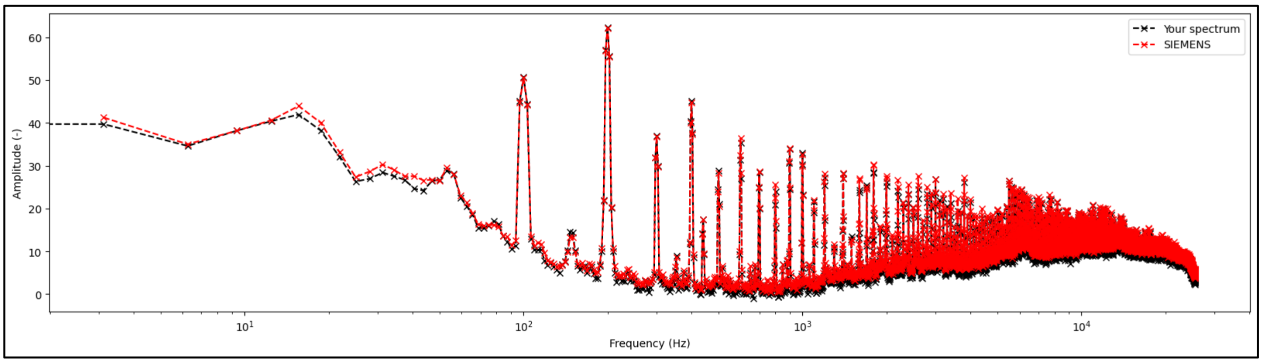

Figure.3 Comparison of SPL spectrum with siemens spectrum

Broadband noise is typically characterized by a relatively constant level of energy across a wide range of frequencies, as opposed to narrowband noise which is concentrated at specific frequencies. From the SPL spectrum above it is identified that the broadband noise is significant in the relatively lower frequency range till 100Hz.

There is a (spike) mode emerged at 100Hz which shouldn't have been there, as according to Eqn.1 the BPF is 200Hz for the 2-blade configuration considered. There can be several reasons for the present of a mode at a frequency other than the BPF. Such as the presence of other sources of noise or vibration in the system that are not directly related to the BPF. For example, there could be mechanical or electrical components in the system that are vibrating at a frequency close to 100 Hz, which is causing a peak in the SPL spectrum at that frequency. It is also possible that the peak is due to artifact or measurement error.

Additionally, there are harmonic components of the BPF at frequencies other than the fundamental BPF (in this case, 200 Hz). Harmonic components are multiples of the fundamental frequency and it indicates that there are non-linearities in the system such as variations in blade shape or blade pitch that create non-uniformities in the flow field around the rotor. These non-uniformities causes variations in the pressure and velocity fields, which in turn generates additional frequency components in the SPL spectrum at harmonic frequencies.